EXCITED HEAVY BARYONS
IN THE BOUND STATE PICTURE

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Abstract

The orbitally excited heavy quark baryons are studied in the Callan Klebanov bound state model with heavy spin symmetry. First, a compact description of the large N_c, infinite heavy quark mass bound state wavefunctions and the collective quantization is given. In order to study the kinematical corrections due to finite masses we motivate an approximate Schrodinger-like equation for the bound state. The effective potential in this equation is compared with the quadratic approximation (spherical harmonic oscillator) to it. This oscillator approximation is seen to be not very accurate. It is noted that the present experimental information cannot be even qualitatively understood with the usual light sector chiral Lagrangian containing only light pseudoscalar mesons. The addition of light vector mesons helps to overcome this problem.

1 Introduction

The "bound state" picture [1, 2], in which a baryon containing a heavy quark is visualized as a bound state of a nucleon-as-Skyrme soliton with a heavy meson, is a very appealing one. It has the interesting feature that experimental information from the mesonic sector of the theory (representing an approximation to QCD in the large N_c limit) can be used to predict the properties of the heavy baryons. Originally it was applied [1, 2] to studying the ordinary hyperons, but a straightforward extension was also made [3] to the c and b baryons. After the recognition of the importance of the Isgur-Wise heavy spin symmetry [4], the study of the c and b baryons was pursued by seveal groups [5]-[10]. The relatively large number of papers suggests the richness and technical complexity of the approach.

The present paper was stimulated by experimental evidence [11] for two orbitally excited heavy $c-$baryons which may be interpreted as the heavy spin multiplet $(\Lambda_c', \Lambda_c'^*)$ with spin-parities $(\frac{1}{2}^-, \frac{3}{2}^-)$. Some properties were already studied in a bound state inspired framework [12] based on a potential for the spatial wavefunction of the form

$$V(r) = V_0 + \frac{1}{2}\kappa r^2 , \tag{1.1}$$

where V_0 and κ are constants. That treatment regarded V_0 and κ as arbitarary parameters to be fit. In the bound state approach they are, however, computable in terms of the Skyrme profile functions and the light meson - heavy meson coupling constants. While the latter especially are by no means precisely fixed it is easy to see that, in models in which the only light mesons present are the pseudoscalars, reasonable choices predict values for V_0 and κ which give an unbound or just barely bound ground state particle Λ_c. (The binding is actually sizeable: $m(\Lambda_c) - m(D) - m(N) = -0.63$ GeV). Hence there appears to be an important gap between the practical use of (1.1) and the actual bound state calculations.

In order to investigate this problem we first formulate an approximate Schrodinger like equation which should hold for finite heavy meson mass M. The underlying equations that one gets, even for the ground state [9], are not of the simple Schrodinger form, but comprise three coupled differential equations. We therefore made a simple approximation which should be good for large M and which gives a Schrodinger-like equation with a potential function which may be compared with (1.1). It turns out that the quadratic approximation (1.1) is roughly reasonable for the low-lying $b-$baryon energies but is significantly worse for the low-lying $c-$baryon energies. The Skyrme potential gives more deeply bound baryons than (1.1); however, the extra binding turns out to be nowhere enough to solve the problem. The non-relativistic form of the approximate Schrodinger equation enables us to easily make very important "two-body" corrections corresponding

to finite nucleon mass (which is infinite in the large N_c starting point) by introducing the reduced mass. It is explained how this further reduces the ground state binding. The most straightforward way to obtain reasonable values of the binding energy seems to be to introduce light vector mesons [7, 8] in addition to light pseudoscalars. These provide a great deal of extra binding strength. The requirement of explaining the spectrum is noted to yield non-trivial constraints on the light vector meson sector of the theory.

Another new aspect of this paper is the presentation of a somewhat streamlined approach to the excited baryon wavefunctions. Following the approach of refs. [7, 8] for the ground state we show how excited state wavefunctions which are already diagonal (in contrast to those of [5] and [10]) may be written down almost by inspection. This is given in section 3 where the highly degenerate spectrum in the $M \to \infty$, $N_c \to \infty$ limit is discussed, including the effects of light vector mesons. The physical states of the theory in this limit are recognized in section 4 after introducing collective variables to describe the Skyrme "tower". This must be quantized as a boson so the low-lying states look like the quark model ones [13] wherein a light diquark (belonging to the flavor $SU(3)$ representations $\bar{\mathbf{3}}$ or $\mathbf{6}$) is rotating around a heavy quark with effective orbital angular momentum ℓ_{eff}. In turn, ℓ_{eff} equals the "light" part of the "grand spin" of the heavy meson in the background Skyrmion field. Finally, the approximate Schrodinger like equation for the physical kinematics situation is treated in section 5 and conclusions are drawn.

2 Notation

We will employ the same notations as in the earlier papers [7] and [8]. For the reader's convenience, a very brief reminder is included here.

The total effective chiral Lagrangian is the sum of a "light" part describing the three flavors u, d, s and a "heavy" part describing the heavy $(0^-, 1^-)$ meson multiplet H and its interaction with the light sector:

$$\mathcal{L}_{eff} = \mathcal{L}_{light} + \mathcal{L}_{heavy} . \tag{2.1}$$

The relevant light fields are the elements of the 3×3 matrix of pseudoscalars ϕ and of the 3×3 matrix of vectors ρ_μ. Some objects which transform simply under the action of the chiral group are

$$\xi = e^{i\phi/F_\pi} , \qquad U = \xi^2 ,$$
$$F_{\mu\nu}(\rho) = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu - i\tilde{g}[\rho_\mu, \rho_\nu] , \tag{2.2}$$

where the pion decay constant $F_\pi \approx 0.132\,\text{GeV}$ and the vector meson coupling constant $\tilde{g} \approx 3.93$. References on $\mathcal{L}_{light}$ may be traced from [7, 8].

The heavy multiplet field combining the heavy pseudoscalar P' and the heavy vector Q'_μ, both moving with a fixed 4-velocity V_μ is given by

$$H = (\frac{1 - i\gamma_\mu V_\mu}{2})(i\gamma_5 P' + i\gamma_\nu Q'_\nu) , \qquad \bar{H} = \gamma_4 H^\dagger \gamma_4 . \tag{2.3}$$

In this convention H has the cannonical dimension one. For $\mathcal{L}_{heavy}$ we take:

$$\frac{\mathcal{L}_{heavy}}{M} = iV_\mu \text{Tr}[H(\partial_\mu - i\alpha\tilde{g}\rho_\mu - i(1-\alpha)v_\mu)\bar{H}] + id\text{Tr}[H\gamma_\mu\gamma_5 p_\mu\bar{H}]$$
$$+ \frac{ic}{m_v}\text{Tr}[H\gamma_\mu\gamma_\nu F_{\mu\nu}(\rho)\bar{H}] , \tag{2.4}$$

where M is the mass of the heavy meson, $m_v \approx 0.77\,\text{GeV}$ is the light vector meson mass and

$$v_\mu , \, p_\mu = \frac{i}{2}(\xi\partial_\mu\xi^\dagger \pm \xi^\dagger\partial_\mu\xi) . \tag{2.5}$$

d, c and $(\alpha\tilde{g})$ are respectively dimensionless coupling constants for the $H-$light pseudoscalar, $H-$light vector magnetic type and $H-$light vector gauge type interactions. It seems fair to say that these coupling constants are not yet precisely fixed from experiment. For definiteness we shall use the values

$$d = 0.53 , \qquad c = 1.6 , \qquad \alpha = 1 . \tag{2.6}$$

The values for d and c are based on single pole fits [14, 15] to the experimental data [16] on the $D \to K$ and $D \to K^*$ semi-leptonic transitions. The ratio c/d is consistent with that obtained [17] using a suitable notion of light vector meson dominance for the $D^* \to D\gamma$ and $D^* \to D\pi$ branching ratios. It should be remarked, however, that some authors have suggested [18] a smaller value of d, around 0.3. The value $\alpha = 1$ is based on light vector meson dominance which [17] seems reasonable. Earlier [8] we used a negative α based on fitting the heavy baryon properties without using the excitation energy constraint. The present paper contains a reconsideration of that fit.

3 Excited Baryon States at the Classical Level

In this section we first write the classical Skyrme solutions for the light part of the action (which is taken to include the first three flavors). Then we find the wavefunctions for which this "baryon as soliton" is bound to the heavy meson H. All the orbitally excited states, rather than just the ground state, will be included. We shall work in the heavy quark symmetry limit so that the radial wavefunctions reduce to their delta-function limits. It

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turns out that the bound excited wavefunctions are remarkably simple generalizations of the ground state one given in [7] and [8] (see section 3 of [8], for example).

The usual hedgehog ansätz for the light pseudoscalars is

$$\xi_c = \begin{pmatrix} \exp[i\hat{\mathbf{x}} \cdot \boldsymbol{\tau} \frac{F(r)}{2}] & 0 \\ 0 & 1 \end{pmatrix} . \tag{3.1}$$

When we include the light vectors, we have similarly the classical solutions

$$\rho_{\mu c} = \begin{pmatrix} \frac{1}{\sqrt{2}}(\omega_{\mu c} + \tau^a \rho^a_{\mu c}) & 0 \\ 0 & 1 \end{pmatrix} , \tag{3.2}$$

with

$$\rho^a_{ic} = \frac{1}{\sqrt{2}\tilde{g}r}\epsilon_{ika}\hat{x}_k G(r) , \quad \rho^a_{0c} = 0 , \quad \omega_{ic} = 0 , \quad \omega_{0c} = \omega(r) . \tag{3.3}$$

The appropriate boundary conditions are

$$F(0) = -\pi , \qquad G(0) = 2 , \qquad \omega'(0) = 0 ,$$
$$F(\infty) = G(\infty) = \omega(\infty) = 0 . \tag{3.4}$$

Now, following the Callan-Klebanov approach, we want to find the wavefunctions of a Schrodinger-like equation for $\bar{H}$ in the classical background field above. Since the above ansätzae mix isospin with orbital angular momentum, it is very convenient to make a partial wave analysis in terms of the grand spin $\mathbf{G}$,

$$\mathbf{G} = \mathbf{I} + \mathbf{L} + \mathbf{S} , \tag{3.5}$$

where $\mathbf{L}$ is the orbital angular momentum of the heavy meson field, $\mathbf{S}$ is its spin and $\mathbf{I}$ its isospin. Due to the heavy spin symmetry, the portion of $\mathbf{S}$ from the heavy quark decouples from the problem. Thus with the decomposition

$$\mathbf{S} = \mathbf{S}' + \mathbf{S}'' , \tag{3.6}$$

where $\mathbf{S}''$ is due to the heavy quark while $\mathbf{S}'$ is the remainder, the truely relevant object is the "light grand spin" :

$$\mathbf{g} = \mathbf{I} + \mathbf{L} + \mathbf{S}' . \tag{3.7}$$

We pointed out earlier [7] that the *ground* state wavefunction is characterized by $g = 0$. Now we want to study the excited states. Remember that in the heavy meson rest frame the 4×4 matrix $\bar{H}$ has non-vanishing elements only in the lower left 2×2 sub-block:

$$\bar{H} = \begin{pmatrix} 0 & 0 \\ \bar{H}^b_{lh} & 0 \end{pmatrix} . \tag{3.8}$$

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Here the index l represents the spin of the light degrees of freedom within the heavy meson, the index h stands for the heavy quark spin and the index b represents the isospin of the light degrees of freedom within the heavy meson. Following [7] and [8] we write the general $g \neq 0$ wavefunction as:

$$\bar{H}^a_{lh}(g, g_3) = \begin{cases} \frac{u(r)}{\sqrt{M}} (\hat{\mathbf{x}} \cdot \boldsymbol{\tau})_{ad} \, \bar{\psi}_{dl}(g, g_3) \, \chi_h \,, & a = 1, 2 \\ 0 \,, & a = 3 \,, \end{cases} \tag{3.9}$$

wherein the radial function $u(r)$ satisfies $r^2 |u(r)|^2 \approx \delta(r)$ and χ_h is the heavy quark spinor. We will see how the presence of the $\hat{\mathbf{x}} \cdot \boldsymbol{\tau}$ factor simplifies the calculations. The remaining factor $\bar{\psi}_{dl}$ is a kind of generalized spherical harmonic with the appropriate covariance properties. There is the standard three-fold ambiguity of which two vectors in (3.7) should be coupled together first. Here, coupling together

$$\mathbf{K} = \mathbf{I} + \mathbf{S}' \tag{3.10}$$

leads to a nice further simplification. Then we write, for a $\bar{\psi}_{dl}$ with orbital angular momentum r and "K-spin" k in (3.10) coupled to light grand spin g:

$$\bar{\psi}_{dl}(g, g_3; r, k) = \sum_{r_3, k_3} C^{r \, k; g}_{r_3 \, k_3; g_3} Y_{r, r_3} \xi_{dl}(k, k_3) \,, \tag{3.11}$$

where C stands for the Clebsch-Gordon coefficients, Y stands for the usual normalized spherical harmonics and $\xi_{dl}(k, k_3)$ are the isospin-light spin wavefunctions:

$$\xi_{dl}(0, 0) = \frac{1}{\sqrt{2}} (s^\uparrow_l i^\downarrow_d - s^\downarrow_l i^\uparrow_d), \qquad \xi_{dl}(1, 1) = s^\uparrow_l i^\uparrow_d \,,$$

$$\xi_{dl}(1, 0) = \frac{1}{\sqrt{2}} (s^\uparrow_l i^\downarrow_d + s^\downarrow_l i^\uparrow_d), \qquad \xi_{dl}(1, -1) = s^\downarrow_l i^\downarrow_d \,, \tag{3.12}$$

s and i being the two component $\mathbf{S}'$ and $\mathbf{I}$ spinors. Notice that multiplying $\bar{\psi}_{dl}$ by $(\hat{\mathbf{x}} \cdot \boldsymbol{\tau})_{ad}$, as in (3.9), does not change its g−spin.

So far, we have just written down possible wave functions allowed by g−spin covariance; the question of which ones or linear combinations actually correspond to bound states has not yet been addressed. To investigate this question we consider the matrix elements of the "potential" V obtained from $\mathcal{L}_{heavy}$ in (2.4). (See (3.2) and (3.3) of [7] for further details). The potential conserves g−spin and parity so we need only its matrix elements between wavefunctions H and H' of the same g and parity:

$$\frac{-M dF'(0)}{2} \int d^3x \, (H')^{a'}_{hl'} (\hat{\mathbf{x}} \cdot \boldsymbol{\tau})_{a'b} \, \boldsymbol{\sigma}_{l'l} \cdot \boldsymbol{\tau}_{bc} \, (\hat{\mathbf{x}} \cdot \boldsymbol{\tau})_{ca} (\bar{H})^a_{lh} + \cdots$$

$$= \frac{dF'(0)}{2} \int d\Omega \, (\bar{\psi}')^*_{bl'} \, \boldsymbol{\sigma}_{l'l} \cdot \boldsymbol{\tau}_{bc} \, (\bar{\psi})_{al} \, (\chi'^\dagger \chi) + \cdots \,, \tag{3.13}$$

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wherein $d\Omega$ denotes the angular integration and the second line follows from the first on the substitution of (3.9) and the observation that the definition $\bar{H} = \gamma_4 H^\dagger \gamma_4$ for the 4-component matrix H results in an extra minus sign for the two component H^a_{hl}. The three dots in (3.13) stand for contributions arising from the couplings of the light vectors to the heavy meson; these do not change our conclusion and will be discussed later.

Notice that all the $\hat{\mathbf{x}} \cdot \boldsymbol{\tau}$ factors have disappeared in the second line of (3.13). This means that the effective potential operator for the wavefunction (3.11) is simply:

$$\frac{1}{2}dF'(0)\,\boldsymbol{\sigma} \cdot \boldsymbol{\tau} = \frac{1}{2}dF'(0)\left[2\mathbf{K}^2 - 3\right] \,, \tag{3.14}$$

where we have used the fact that $\boldsymbol{\sigma} \cdot \boldsymbol{\tau}$ is acting on wavefunctions of definite $\mathbf{K}$. For $k = 0$ we have the energy eigenvalue $-\frac{3}{2}dF'(0) \approx -0.63$ GeV, which indicates that the $k = 0$ states are the bound ones. The (in general) three $k = 1$ states (corresponding to r in (3.11) taking on the values $g - 1$, g, and $g + 1$) have the energy eigenvalue $+\frac{1}{2}gF'(0)$ and are unbound in this model. The wavefunctions $\bar{\psi}_{dl}$ in (3.11) as well as the $\bar{H}$'s in (3.9) are already diagonal - a circumstance following from the choice of k as the intermediate label for constructing states of good $\mathbf{g}$. It is not actually necessary to carry out the multiplication by $\hat{\mathbf{x}} \cdot \boldsymbol{\tau}$ in (3.11); however, this is done in Appendix A. We shall refer to the "$k-$value" of a wavefunction as that of the factor $\bar{\psi}_{dl}$ in (3.11). As seen in Appendix A, this gets modified on multiplication by $\hat{\mathbf{x}} \cdot \boldsymbol{\tau}$.

The bound state wavefunctions may be written in a very simple form. Since they have $k = 0$, (3.11) simplifies to $Y_{gg_3}\,\xi_{dl}(0,0)$. Furthermore $\xi_{dl}(0,0) = \frac{1}{\sqrt{2}}\epsilon_{dl}$, giving finally for (3.9):

$$\bar{H}^a_{lh}(g, g_3, s''_3) = \begin{cases} \frac{u(r)}{\sqrt{2M}}(\hat{\mathbf{x}} \cdot \boldsymbol{\tau})_{ad}\,\epsilon_{dl}\,Y_{gg_3}\,\chi_h \,, & a = 1, 2 \\ 0 \,, & a = 3 \,. \end{cases} \tag{3.15}$$

The $g = 0$ ground state wave function is seen, using $Y_{00} = 1/\sqrt{4\pi}$, to coincide with (3.7) of [8]. In fact, the bound orbitally excited wavefunctions are simply obtained by multiplying the ground state one by $\sqrt{4\pi}Y_{gg_3}$.

The parities of the bound state wavefunctions are given by the formula:

$$\text{parity} = (-1)^g \,. \tag{3.16}$$

This follows most directly from the fact that a negative parity meson is being bound in a linear combination of states with orbital angular momenta $g - 1$ and $g + 1$ (see the first line of (A1)). Of the three (in general) unbound states with given g, one has parity $(-1)^g$ and the other two have parity $-(-1)^g$.

The light grand spin quantum number g has yet further physical significance for the bound state wavefunctions. Note that in the extreme limit in which we are presently

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working all the bound states are degenerate in energy. This degeneracy will be broken if one allows for finite masses. We then expect a centrifugal contribution to the energy of the form

$$\frac{1}{2Mr^2}\ell_{eff}(\ell_{eff}+1) \, , \tag{3.17}$$

which would raise the energies for orbital excitations. Callan and Klebanov pointed out in their original paper [1] that ℓ_{eff} differs from the orbital angular momentum due to the isospin-angular momentum mixing in the Skyrme ansätz. It turns out that, in fact,

$$\ell_{eff} = g \, . \tag{3.18}$$

To see this, note that the centrifugal energy operator is approximately at small r given by

$$\frac{1}{2Mr^2}(\mathbf{L}^2 + 2 + 4\mathbf{I}\cdot\mathbf{L}) \, , \tag{3.19}$$

as is mentioned in [1] and as also emerges in the approximation for finite M which perserves the heavy quark spin multiplets (to be discussed in Section 5 of this paper). With $\boldsymbol{\lambda} = \mathbf{L} + \mathbf{I}$, (3.19) may be rewritten as $(2\boldsymbol{\lambda}^2 - \mathbf{L}^2 + \frac{1}{2})/(2Mr^2)$. Now the bound state wavefunction is a linear combination of $\ell = g - 1$ and $\ell = g + 1$ pieces. Considering a basis in which $\boldsymbol{\lambda}$ is diagonal, we note that the $\ell = g - 1$ piece can only couple to light grand spin g for the choice $\lambda = g - \frac{1}{2}$. Then the centrifugal energy becomes

$$\frac{1}{2Mr^2}\left[2(g-\frac{1}{2})(g+\frac{1}{2}) - (g-1)g + \frac{1}{2}\right] = \frac{1}{2Mr^2}g(g+1)\,. \tag{3.20}$$

Similarly the $\ell = g + 1$ piece requires $\lambda = g + \frac{1}{2}$, which leads to the same result.

To end this section we give the form of the potential operator when the effects of light vector mesons are included. Then (3.14) should be replaced by:

$$\boldsymbol{\sigma}\cdot\boldsymbol{\tau}\left[\frac{1}{2}dF'(0) - \frac{c}{m_v\tilde{g}}G''(0)\right] + \mathbf{1}\left[-\frac{\alpha\tilde{g}}{\sqrt{2}}\omega(0)\right] \, . \tag{3.21}$$

In this formula d, c and α are the heavy meson$-$light meson coupling constants defined in (2.4), while $F(r)$, $G(r)$ and $\omega(r)$ are the pseudoscalar, ρ meson and ω meson soliton profile functions defined in (3.1)$-$(3.4). Notice that the ρ meson piece has the same $\boldsymbol{\sigma}\cdot\boldsymbol{\tau}$ factor as the pseudoscalar piece while the ω meson piece has simply a $\mathbf{1}$ in the light spin$-$isospin spaces. This shows that the wavefunctions of (3.9) and (3.11) are still diagonal. The eigenvalues for the $k = 0$ states and for the $k = 1$ states are now read off to be:

$$\begin{aligned} V(k=0) &= -\frac{3}{2}dF'(0) + \frac{3c}{m_v\tilde{g}}G''(0) - \frac{\alpha\tilde{g}}{\sqrt{2}}\omega(0)\,, \\ V(k=1) &= \frac{1}{2}dF'(0) - \frac{c}{m_v\tilde{g}}G''(0) - \frac{\alpha\tilde{g}}{\sqrt{2}}\omega(0)\,. \end{aligned} \tag{3.22}$$

The quantitites $F'(0)$, $G''(0)$ and $\omega(0)$ are obtained by solving the coupled differential equations which arise by minimizing the static energy of $\mathcal{L}_{light}$ (describing the ordinary baryons). This yields [19] for a typical best fit to baryon and meson masses:

$$F'(0) = 0.795\,\text{GeV}, \qquad G''(0) = -0.390\,\text{GeV}, \qquad \omega(0) = -0.094\,\text{GeV}\,. \qquad (3.23)$$

It should be remarked [20] that the sign of $\omega(0)$ is linked to the sign of the $\rho\omega\phi$ coupling constant; the sign has been chosen [19, 20] to yield a best fit to light baryon electromagnetic form factors. To be on the conservative side one might want to consider the possibility of reversing this sign. Now, substituting (2.6) and (3.23) into (3.22) yields

$$\begin{aligned} V(k=0) &= -0.63 - 0.62 + 0.26\,(-0.26) = -0.99\,(-1.51)\,\text{GeV}, \\ V(k=1) &= +0.31 + 0.21 + 0.26\,(-0.26) = +0.78\,(0.26)\,\text{GeV}, \end{aligned} \qquad (3.24)$$

wherein the order of the terms in (3.22) has been retained. The numbers in parentheses correspond to reversing the sign of $\omega(r)$ in 3.23. We may observe that the ρ meson contribution ($G''(0)$ terms) strengthens the attraction in the bound channel and also increases the repulsion in the unbound channel. The ω term is not dominant but may have significant effects. The binding in the present approximation seems somewhat too large. However, finite M effects will provide a substantial reduction.

4 Physical States of the Model

The states of definite angular momentum and isospin emerge, in the soliton approach, after collective quantization. The collective angle-type variable $A(t)$ [21, 22] is introduced on the light fields as

$$\xi(\mathbf{x}, t) = A(t)\xi_c(\mathbf{x})A^\dagger(t), \qquad \rho_\mu(\mathbf{x}, t) = A(t)\rho_{\mu c}(\mathbf{x})A^\dagger(t)\,, \qquad (4.1)$$

where $\xi_c(\mathbf{x})$ and $\rho_{\mu c}(\mathbf{x})$ are given in (3.1)-(3.3) and generalized angular velocities Ω_k are defined by

$$A^\dagger \dot{A} = \frac{i}{2}\sum_{k=1}^{8} \lambda_k \,\Omega_k\,, \qquad (4.2)$$

the λ_k being $SU(3)$ Gell-Mann matrices. All the heavy meson states, rather than just the ground state, may be included by introducing a Fock representation for the field $\bar{H}$ as

$$\bar{H}(\mathbf{x}, t) = \sum_n \bar{H}_n(\mathbf{x})\, e^{iE_n t}\, a_n^\dagger\,, \qquad (4.3)$$

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wherein the stationary states $n = \{g, g_3; r, k, s_3''\}$ are given in (3.9) and (3.11) (the unbound states are also included) while $a_n^\dagger$ is the creation operator for the state n with energy eigenvalue E_n. The "light part" of $\bar{H}$ is subjected to a collective rotation with the replacement

$$\bar{H}_{hl}^a(\mathbf{x}, t) \to A_{ab}(t) \bar{H}_{hl}^b(\mathbf{x}, t) \,. \tag{4.4}$$

Now the collective Lagrangian, L_{coll}, is obtained by substituting (4.1) and (4.4) into $\int d^3x (\mathcal{L}_{light} + \mathcal{L}_{heavy})$ and carrying out the spatial integration. The final physical states are recognized after the quantization of L_{coll}. There are two ways in which this L_{coll} differs from that of the usual $SU(3)$ Skyrme model [22]. The first is that instead of a term $-\frac{\sqrt{3}}{2} \Omega_8$ there is now (in the one heavy quark subspace) a term $-\frac{\sqrt{3}}{3} \Omega_8$. As discussed in section 4 of [8], the physical significance of this fact is that there now exists a constraint on the quantum states which requires the "Skyrmion rotator" to transform as an irreducible representation $\{\mu\}$ of $SU(3)$ which contains a state with hypercharge $Y = 2/3$. This particular state must necessarily have integer isospin. Furthermore its spin, denoted $\mathbf{J}_s$, must equal its isospin. The two lowest Skyrmion multiplets (higher ones are probably model artifacts) are

$$(\mu = \bar{\mathbf{3}}, \quad J_s = 0)$$
$$(\mu = \mathbf{6}, \quad J_s = 1). \tag{4.5}$$

Note that the Skyrmion rotator behaves as a boson; at the collective level it evidently describes a diquark state. This analysis is forced upon us when we consider the case of three light flavors [23] with the attendant Wess-Zumino term. It agrees with the original picture [1] of "spin-isospin" transmutation; namely, at the collective level the heavy meson field

$$\text{(i) loses its flavor quantum numbers}$$
$$\text{(ii) acquires a spin equal to } \mathbf{G} \tag{4.6}$$

The full baryon state is a product of the bosonic diquark rotator in (4.5) and the fermionic bound state wavefunction corresponding to (4.6).

The second way in which L_{coll} differs from that of the $SU(3)$ Skyrme model is the presence of the following term linear in $\boldsymbol{\Omega}$:

$$M \int d^3x \, \Gamma_j(\Omega) \, H_{hl}^a \, (\tau^j)_{ab} \, \bar{H}_{lh}^b,$$
$$\Gamma_j(\Omega) = (\frac{1}{2} - \alpha)\Omega_j - (1 - \alpha)\, \hat{\mathbf{x}} \cdot \boldsymbol{\Omega} \, \hat{x}_j \,, \tag{4.7}$$

where $\bar{H}$ is taken from (4.3) and $\Gamma_j(\Omega)$ is evaluated near the origin of $\mathbf{x}$-space. From the analysis of [1], one might expect a term like (4.7) to lead to (hyperfine) splitting of the baryons belonging to a given heavy spin multiplet. Fortunately, this does not happen as one may see by inspection using the bound (negative energy) wavefunctions of (3.15). Then (4.7) contains a factor:

$$\epsilon_{d'l}\,(\hat{\mathbf{x}}\cdot\boldsymbol{\tau})_{d'a}\,(\tau^j)_{ab}\,(\hat{\mathbf{x}}\cdot\boldsymbol{\tau})_{bd}\,\epsilon_{dl} = \mathrm{Tr}(\hat{\mathbf{x}}\cdot\boldsymbol{\tau}\,\tau^j\,\hat{\mathbf{x}}\cdot\boldsymbol{\tau}) = 0\,. \tag{4.8}$$

Thus, in the heavy mass $M \to \infty$ limit, the bound eigenstates of the collective Hamiltonian are simply products of the wavefunctions (3.15) with the diquark Skyrme rotator wavefunctions. Taking account of (4.6 ii), the total angular momentum $\mathbf{J}$ is given by

$$\mathbf{J} = \mathbf{J}_s + \mathbf{G} = \mathbf{J}_s + \mathbf{g} + \mathbf{S}''\,, \tag{4.9}$$

where, for convenience, the heavy quark spin $\mathbf{S}''$ has been made explicit in the last step. The Skyrme rotator wavefunctions are [24]:

$$\Psi_{rot}(\mu, YII_3, J_sM) = (-1)^{J-J_3}\sqrt{dim\mu}\ D^{(\mu)*}_{Y,I,I_3;\frac{2}{3},\,J_s,\,-M_s}(A)\,, \tag{4.10}$$

where $D^{(\mu)}(A)$ is the representation matrix of $SU(3)$. Defining the total light spin $\mathbf{j} = \mathbf{J} - \mathbf{S}''$, we write the overall wavefunctions for the bound states as:

$$\Psi^*(\mu; Y, I, I_3; J_s, g, j; j_3, s_3'') = \sum_{M,g_3} C^{J_s,g;j}_{M,g_3;j_3}\,\Psi^*_{rot}(\mu, YII_3, J_sM)\,\bar{H}^a_{lh}(g, g_3, s_3'')\,, \tag{4.11}$$

where $\bar{H}^a_{lh}$ is given in (3.15). Finally $\mathbf{j}$ and $\mathbf{S}''$ may be added using $C^{j,\frac{1}{2};J}_{j_3,s_3;J_3}$ to yield the heavy spin baryon multiplets having $J = j \pm \frac{1}{2}$.

All the states in (4.11) have, up to relatively small $O(1/N_c)$ corrections, the degenerate energy eigenvalues $V(k=0)$ in (3.22). The $O(1/N_c)$ corrections associated with the present collective quantization splits the $\bar{\mathbf{3}}$ and $\mathbf{6}$ $SU(3)$ representation states according to [8]:

$$M(\mathbf{6}) - M(\bar{\mathbf{3}}) = \frac{2}{3}\,[m(\Delta) - m(N)]\,, \tag{4.12}$$

where Δ and N stand for the $\Delta(1230)$ and nucleon masses. Now let us ennumerate the physical states. We expect, as mentioned after (3.17), that finite M will split the huge degeneracy so that $g = 0$ corresponds to the ground states, $g = 1$ to the first excited states and so on. For $g = 0$ the Clebsch Gordon addition in (4.11) becomes trivial and the discussion reduces to that given in section 4 of [8]. All the $g = 0$ states have (see (3.16)) positive parity. The $SU(3)$ representation $\bar{\mathbf{3}}$ with $j = J_s = 0$ has the content

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$\{\Lambda_Q, \Xi_Q(\bar{\mathbf{3}})\}$. It has spin $J = \frac{1}{2}$ obtained by adding the heavy spin $s'' = \frac{1}{2}$ to $j = 0$. The $SU(3)$ representation $\mathbf{6}$ has $j = J_s = 1$. Adding s'' to this yields a degenerate heavy spin multiplet with both $J = \frac{1}{2}$ and $J = \frac{3}{2}$ members. The flavor content is denoted $\{\Sigma_Q, \Xi_Q(\mathbf{6}), \Omega_Q\}$ and $\{\Sigma_Q^*, \Xi_Q^*(\mathbf{6}), \Omega_Q^*\}$.

It is sufficient to consider the first orbitally excited states with $g = 1$ in order to see the general pattern. All the $g = 1$ states have, according to (3.16), negative parity. The $SU(3)$ $\bar{\mathbf{3}}$ diquark rotator state with $J_s = 0$ is coupled in (4.11) to total light spin $j = g = 1$. Combining this, in turn, with the heavy quark spin $\mathbf{S}''$ yields a heavy spin multiplet with both $J = \frac{1}{2}$ and $J = \frac{3}{2}$ members. These are denoted as $\{\Lambda_Q', \Xi_Q'(\bar{\mathbf{3}})\}$ and $\{\Lambda_Q'^*, \Xi_Q'^*(\bar{\mathbf{3}})\}$. The $SU(3)$ $\mathbf{6}$ diquark rotator state with $J_s = 1$ is coupled in (4.11) to total light spin $j = 0, 1 \,\text{or}\, 2$. Combining each of these three with $\mathbf{S}''$ yields three heavy spin multiplets with total spin contents $(J = \frac{1}{2})$, $(J = \frac{1}{2}, J = \frac{3}{2})$ or $(J = \frac{3}{2}, J = \frac{5}{2})$. For each of these there are 6 flavor states analogous to $\{\Sigma_Q, \Xi_Q(\mathbf{6}), \Omega_Q\}$. Notice that the zero isospin states denoted Λ_c' and $\Lambda_c'^*$ are possible candidates for recently discovered [11] resonances.

$SU(3)$ splittings in the $M \to \infty$ limit have been discussed in section 5 of [8] for the $g = 0$ states; a similar analysis can be given for the orbitally excited states.

The positive energy (unbound) states, corresponding to $k = 1$ in (3.11), can be ennumerated in a similar way using the appropriate $\bar{H}_{lh}^a(g, g_3; r, k, s_3'')$ in (4.11). While these states would presumably not be bound in the true theory, they can be expected to play a role as virtual intermediate states for further application of the formalism.

5   Kinematic Corrections

The preceeding results hold in the large N_c limit (where the nucleon mass is formally infinite) and in the large M limit (where the heavy meson mass is formally infinite). In order to compare with experiment it is clearly important to get an idea of the corrections to be expected in the real world. A complete discussion of these effects would be enormously complicated and beyond the scope of the present investigation. Thus we will content ourselves with a somewhat schematic model which has the advantage of simplicity and which is familiar enough to stimulate our intuition. The most straightforward way to proceed is to start with the simplest "ordinary field" Lagrangian which reduces to the heavy field Lagrangian (2.4) in the $M \to \infty$ limit. Such an ordinary field Lagrangian, constructed with a heavy pseudoscalar $SU(3)$ triplet P and a heavy vector $SU(3)$ triplet Q_μ was given in (3.25) of [14] in connection with our original discussion of (2.4). For additional simplicity we shall at first neglect the heavy meson interactions with the light

vectors, although we expect from (3.22) and (3.24) that they are actually important. Then the heavy Lagrangian becomes:

$$\mathcal{L}_{heavy} = \quad -\mathcal{D}_\mu P \mathcal{D}_\mu \bar{P} - M^2 P \bar{P} - \frac{1}{2} \left(\mathcal{D}_\mu Q_\nu - \mathcal{D}_\nu Q_\mu \right) \left(\mathcal{D}_\mu \bar{Q}_\nu - \mathcal{D}_\nu \bar{Q}_\mu \right) - M^{*2} Q_\mu \bar{Q}_\mu$$
$$+2iMd \left(P p_\mu \bar{Q}_\mu - Q_\mu p_\mu \bar{P} \right) - id' \epsilon_{\beta\alpha\rho\mu} \left(\mathcal{D}_\rho Q_\alpha p_\mu \bar{Q}_\beta - Q_\alpha p_\mu \mathcal{D}_\rho \bar{Q}_\beta \right) , \quad (5.1)$$

where $\mathcal{D}_\mu \bar{P} = \partial_\mu \bar{P} - iv_\mu \bar{P}$, etc. Note that v_μ and p_μ are defined in (2.5). In writing (5.1) we have allowed for different P and Q_μ masses M and M^* as well as different coupling constants d and d'. But for simplicity we shall further restrict $M = M^*$ and $d = d'$; thus our model will treat only those corrections to the heavy quark symmetry due to finite M. The same model (5.1) has been investigated by Oh et. al. [9] for the ground state heavy baryons and with choices of $d \neq d'$ and $M \neq M'$ made to fit the heavy baryon masses with experiment. Our treatment will differ in a number of ways. First we shall approximate the three coupled differential equations which result by a simple Schrodinger-like equation. We shall generalize the approximate equation to include excited states, and compare with the more standard picture [12] of a quadratic potential. We will also give some discussion of corrections due to the finite nucleon mass.

To begin, we look for the stationary ground state solutions of the equations of motion (see Appendix B) which result from (5.1). We know what this should look like in the heavy limit. Remember that the field P above is related to the field P' in (2.3) by

$$P = e^{imV \cdot x} P' , \quad (5.2)$$

and similarly for Q_μ. The ground state ($g = 0$) wavefunction for $\bar{H}$ in (3.15) translates to

$$\bar{P}'^b = \frac{-i}{2} \text{Tr}(\gamma_5 \bar{H}) = \frac{i}{2} \frac{u(r)}{\sqrt{8\pi M}} (\hat{\mathbf{x}} \cdot \boldsymbol{\tau})_{bd} \rho_d ,$$
$$\bar{Q}'^b_j = \frac{-i}{2} \text{Tr}(\gamma_j \bar{H}) = \frac{1}{2} \frac{u(r)}{\sqrt{8\pi M}} \left[\hat{x}_j \delta_{bd} - i(\hat{\mathbf{x}} \times \boldsymbol{\tau}_{bd})_j \right] \rho_d , \quad (5.3)$$

where b is the 2-valued isospin index and $\rho_d = \epsilon_{de} \chi_e$. Taking (5.2) into account and allowing different radial dependences for the different terms in (5.3) suggests the ansätzae [9] for the ordinary (unprimed) field stationary solutions:

$$\bar{P}^b = \frac{\phi(r)}{\sqrt{4\pi}} (\hat{\mathbf{x}} \cdot \boldsymbol{\tau})_{bd} \rho_d e^{i\omega t} ,$$
$$\bar{Q}^b_j = \frac{1}{\sqrt{4\pi}} \left[i\hat{x}_j \delta_{bd} \psi_1(r) + \frac{1}{\sqrt{2}} (\hat{\mathbf{x}} \times \boldsymbol{\tau}_{bd})_j \psi_2(r) \right] \rho_d e^{i\omega t} , \quad (5.4)$$

where ω is the relativistic energy. At the ordinary field level we should interpret ρ_d as the isospace spinor. With this parametrization the $M \to \infty$ limit is expressed as

$$\phi = -\psi_1 = -\frac{\psi_2}{\sqrt{2}} . \quad (5.5)$$

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Substituting (5.4) into the $\bar{P}$ equation of motion given in (B.1) yields the differential equation:

$$\phi'' + \frac{2}{r}\phi' + \left[\omega^2 - M^2 + \frac{1}{r^2}\{-2 + \frac{1}{2}(1 - \cos F)(3 + \cos F)\}\right]\phi - MdF'\psi_1$$
$$+ \frac{\sqrt{2}Md\sin F}{r}\psi_2 = 0 , \tag{5.6}$$

where a prime denotes differentiation with respect to the radial coordinate r. The two other similar coupled equations which result from (B.2) are given in (B.4) and (B.5). First let us check the consistency of the $M \to \infty$ limit. Dropping all terms not proportional to M and substituting (5.5) we easily see that each of (5.6), (B.4) and (B.5) reduces to

$$\left[(\omega^2 - M^2) + Md(F' - \frac{2\sin F}{r})\right]\phi(r) = 0 . \tag{5.7}$$

This equation has a solution where $\phi(r)$ is sharply peaked around the origin and where

$$E_{eff} = -\frac{1}{2}d\left[F' - \frac{2\sin F}{r}\right]_{r=0} = -\frac{3}{2}dF'(0) . \tag{5.8}$$

Here we have defined

$$E_{eff} = \frac{\omega^2 - M^2}{2M} \approx \omega - M . \tag{5.9}$$

(5.8) is seen to agree with the binding energy $V(k=0)$ in (3.22), when the contributions from the light vectors are neglected.

Now let us approximate (5.6) to achieve an easy form for convenient further study. It seems very natural to isolate the effects of finite M by retaining it in (5.6) while approximating ψ_1 and ψ_2 by their $M \to \infty$ forms from (5.5). Then we end up with a completely standard looking Schrodinger equation for the radial wavefunction. We set $\phi(r) = w(r)/r$ as usual, divide through by $2M$ and finally add the "centrifugal" term (3.17) so as to generalize the equation to include orbitally excited partial waves. Then (5.6) becomes

$$-\frac{1}{2M}w''(r) + \left[\frac{\ell_{eff}(\ell_{eff}+1)}{2Mr^2} + V_{eff}(r)\right]w(r) = E_{eff}\,w(r) , \tag{5.10}$$

wherein,

$$V_{eff}(r) = \frac{-d}{2}\left(F' - \frac{2\sin F}{r}\right) + \frac{1}{Mr^2}\left[1 - \frac{1}{4}(1 - \cos F)(3 + \cos F)\right] . \tag{5.11}$$

Of course, $w(r)$ has an implicit ℓ_{eff} label. Even though (5.10) looks exactly like a non-relativistic Schrodinger equation it actually (noting (5.9)) contains the energy in the

characterestic relativistic manner, $(\omega^2 - M^2)$. It describes a meson of mass M in the potential field $V_{eff}(r)$ due to an infinitely heavy (large N_c limit) nucleon. Notice that the $\frac{1}{M}$ piece in V_{eff} which has an overall $\frac{1}{r^2}$ factor behaves as r^2 for small r; this term is in fact very small. There is no exact analytic form for the Skyrme profile $F(r)$. To make the equation self-contained we adopt the Atiyah-Manton approximation [25]:

$$F(r) \approx -\pi \left[1 - \frac{r}{(\lambda^2 + r^2)^{1/2}} \right] , \qquad (5.12)$$

where the parameter choice $\lambda^2 = 15.61584 \, \text{GeV}^{-2}$ corresponds to $F'(0) = 0.795 \, \text{GeV}$ from (3.23). Using (5.12), the effective potential $V_{eff}(r)$ is graphed in Fig. 1. Also shown is the quadratic approximation which leads to a spherical harmonic oscillator potential:

$$V_{SHO} = -\frac{3}{2} dF'(0) + \frac{1}{2}\kappa r^2 , \qquad (5.13)$$

where κ is given in (4.4) of [7]. With the neglect of the light vectors and the choice (3.23), we have $\kappa = 0.1562 \, \text{GeV}^3$. It is seen that the quadratic approximation is not a very accurate representation away from small r. This region is especially relevant for orbitally excited states. On the other hand we note that, as expected, the $V_{eff}(r)$ which results from the Skyrme approach is not confining. Thus it is probably not trustworthy for higher orbitally excited states.

To avoid confusion we remind the reader that, according to the discussion of section 4, there are many physical states which correspond to a given ℓ_{eff}. They all have parity $(-1)^{\ell_{eff}}$. The heavy baryons which belong to the $SU(3)$ $\mathbf{3}$ representations comprise a heavy spin multiplet with spin content $(\ell_{eff} - \frac{1}{2})$, $(\ell_{eff} + \frac{1}{2})$ [for $\ell_{eff} = 0$ this collapses to just $\frac{1}{2}$]. The heavy baryons which belong to the $SU(3)$ $\mathbf{6}$ representation form three heavy spin multiplets with spin contents $[(\ell_{eff} - \frac{3}{2}), (\ell_{eff} - \frac{1}{2})]$, $[(\ell_{eff} - \frac{1}{2}), (\ell_{eff} + \frac{1}{2})]$ and $[(\ell_{eff} + \frac{1}{2}), (\ell_{eff} + \frac{3}{2})]$ [for $\ell_{eff} = 0$ this collapses to just $(\frac{1}{2}, \frac{3}{2})$].

Now let us consider the numerical solutions of (5.10)- (5.12). For definiteness we take the weighted average of the pseudoscalar and vector meson masses to obtain M; this yields $M = 1.94 \, \text{GeV}$ for the charmed meson mass and $M = 5.314 \, \text{GeV}$ for the $b-$meson mass. The wavefunctions $w(r)$ are taken to behave as $r^{(\ell_{eff}+1)}$ at the origin. Displayed in Table 1 are the values of E_{eff} corresponding to the most deeply bound state in each of the $\ell_{eff} = 0, 1, 2$ channels for both the $c-$baryons and $b-$baryons. For comparison we also show the results obtained from the spherical harmonic oscillator (SHO) approximation (5.13). In that case we have a well known analytic formula for the energy levels:

$$E_{eff}^{(N)} = -\frac{3}{2} dF'(0) + \sqrt{\frac{\kappa}{M}} \left(\frac{3}{2} + N \right) , \qquad (5.14)$$

ℓ_{eff}	E_{eff} from (5.10)	SHO approximation
	$c-$baryons	
0	-0.277	-0.204
1	-0.113	$+0.079$
2	-0.012	$+0.363$
	$b-$baryons	
0	-0.403	-0.373
1	-0.277	-0.205
2	-0.169	-0.031

Table 1: E_{eff} in GeV from (5.10) and from the spherical harmonic oscillator (SHO) approximation.

where the level N corresponds here to states with $\ell_{eff} = N$, $N - 2$, $\cdots$. It is interesting to note that the exact numerical ground state ($\ell_{eff} = 0$) energy eigenvalues are more negative than the approximate SHO ones. This can be physically understood from Fig. 1, since the SHO potential is narrower and should thus have more zero point quantum fluctuation energy. As we expect, the SHO approximation is better for the $b-$baryon case than for the lighter $c-$baryon case where the more spread out wavefunctions sample more of the differing large r regions. Similarly, the SHO approximation is better for the ground state than for the excited states. In fact the $c-$baryon $\ell_{eff} = 1$ state is unbound in the SHO approximation. The model also predicts radially excited wavefunctions. For the $c-$baryon another $\ell_{eff} = 0$ level is found at -0.051 GeV. In the SHO approximation this is expected to be degenerate with the unbound $\ell_{eff} = 2$ level at $+0.363$ GeV. As $M \to \infty$ the SHO approximation results and the Skyrme potential results get closer to each other. For example, already at $M = 15$ GeV the difference between the predicted values of the ground state E_{eff} differ only by 1.5%. However, the collapse to the $M \to \infty$ value of -0.63 GeV is rather slow; even at $M = 100$ GeV, there is a 10% difference. Note that the finite M effects are large *both* for the c and b baryons.

A crucial question, of course, is how well these results agree with experiment. For this purpose it will be seen to be sufficient to neglect the relatively small $1/N_c$ corrections (of order 0.1 GeV). Then the data at present yields just three relevant numbers. First there are the "binding energies"

$$
\begin{aligned}
\text{B.E.}_c &= m(\Lambda_c) - m(N) - m(D) = -0.63\,\text{GeV}\,, \\
\text{B.E.}_b &= m(\Lambda_b) - m(N) - m(B) = -0.78\,\text{GeV}\,.
\end{aligned}
\tag{5.15}
$$

In addition, if the recently discovered [11] zero isospin heavy baryons are identified with the Λ'_c and Λ'^*_c mentioned in section 4, we have the excitation energy

$$
\text{E.E.}_c = m(\Lambda'_c) - m(\Lambda_c) = 0.31\,\text{GeV}\,.
\tag{5.16}
$$

It seems natural to identify the binding energy with E_{eff}. Then the comparison with experiment is presented in Table 2. In column (A) the $M \to \infty$ results are given. The agreement with the binding energies is reasonably good but the excitation energies are predicted to be zero. The results of taking finite M corrections into account via the numerical solution of (5.10)-(5.12) are shown in column (B). We note that the general trend of the experimental data is reproduced in the sense that each prediction of the model is about half of the experimental value. The experimentally greater binding for the b baryons compared to the c baryons is reproduced. This picture is very suggestive since we have been, for simplicity, working in a model which does not include the light vector mesons. Equations (3.22) and (3.24) show that one may expect the binding to be greatly strengthened by the addition of the light vectors. To test this roughly one may use the SHO approximation including light vectors. Taking the inputs from (2.6) and (3.23) we have, in addition to $V(k = 0) = -0.99\,(-1.51)\,\text{GeV}$ from (3.24), $\kappa = 0.295\,(0.375)\,\text{GeV}^3$ from (4.4) of [7], resulting in

$$V_{SHO}\,(\text{with light vectors}) = -0.99\,(-1.51) + 0.295\,(0.375)\frac{r^2}{2} , \qquad (5.17)$$

where the numbers in parentheses correspond to the reversed sign for $\omega(r)$. (Of course it is equivalent, from the present standpoint, to keep the sign of $\omega(r)$ and reverse the sign of α.)

The predictions from this model are shown in column (C) and are seen to be significantly closer to experiment. (The values in parentheses show the sensitivity to changing the sign of $\omega(r)$.) It thus appears that the Callan Klebanov approach including light vector mesons and $\frac{1}{M}$ corrections can roughly explain the general features of the presently observed heavy baryon mass spectrum. This picture is however achieved in the $N_c \to \infty$ limit in which the nucleon mass is formally infinite. From a naive kinematical point of view this seems peculiar, although we may perhaps argue that the $N_c \to \infty$ limit is often more accurate than it has a right to be.

It is clearly of interest to explore the "two body" corrections corresponding to taking the nucleon mass to be its finite experimental value. The most straightforward way to proceed is to note that (5.10) may be regarded as a non-relativistic Schrodinger equation with E_{eff} the binding energy. Then we replace M in (5.10) with the reduced mass $\mu = (M\,m(N))/(M + m(N))$ according to the usual prescription. If this is applied to finding the ground state $\ell_{eff} = 0$ energy for the heavy charmed baryon (where $\mu = 0.633$ GeV) we find $E_{eff} = -0.084$ GeV rather than the value -0.277 GeV listed in Table 1. This drastic reduction in binding strength can be easily understood in the SHO approximation. There the classical binding energy of -0.63 GeV is pushed up by the zero point energy

	Expt.	(A)	(B)	(C)	(D)
B.E.$_c$	−0.63	−0.63	−0.277	−0.41 (−0.85)	+0.31 (−0.36)
B.E.$_b$	−0.78	−0.63	−0.403	−0.64 (−1.11)	−0.08 (−0.48)
E.E.$_c$	0.31	0	0.164	0.39 (0.44)	0.68 (0.77)
E.E.$_b$	?	0	0.126	0.24 (0.27)	0.61 (0.69)

Table 2: Comparison with experiment in the large N_c limit. All quantities are in GeV. Column (A) gives the prediction in the $M \to \infty$ limit, column (B) corresponds to the solutions of (5.10)-(5.12). Column (C) corresponds to the SHO approximation when light vectors are included. Column (D) further includes N_c subleading two-body corrections due to finite nucleon mass to the approximation (C).

$\frac{3}{2}\sqrt{\frac{\kappa}{\mu}}$. However the replacement $M \to \mu$ greatly increases the zero point energy.

To get a somewhat more realistic impression we use the SHO approximation with inclusion of the light vectors (5.17). Then we obtain for the binding and excitation energies the results in column (D) of Table 2. It is clear that choosing the solution in parentheses, corresponding to the choice of positive $\omega(0)$ in (3.23), yields a fairly reasonable picture for the ground state energies. The binding energies are somewhat too small in magnitude but we have seen already for the model without light vectors (Table 1) that there is more binding for the c−baryons with the Skyrme profile than is indicated by the SHO approximation. Thus we expect some improvements when we consider, in the future, the analog of (5.10) with the inclusion of light vectors in $V_{eff}(r)$. Similarly, we expect considerable reductions in the predicted excitation energies. More precise determinations of the coupling constants d, c and α is evidently also important. Furthermore, relativistic two body effects may play a role.

It does not seem that an adequate description of the spectrum can be obtained in a model with just the light pseudoscalars included and when finite $m(N)$ is taken into account - the lowest lying charmed baryon would be barely bound.

In this paper we have presented a streamlined formalism for investigating, in the Callan Klebanov bound state picture, the orbitally excited baryons containing a heavy quark. The effects of light vector mesons and strange quarks were included. Wavefunctions which diagonalize the bound state Hamiltonian in the $M \to \infty$, $N_c \to \infty$ limit were obtained in a transparent way. These correspond to binding energies which are somewhat too large. Furthermore there is no splitting between any excited states and the ground state. For the purpose of investigating the more realistic case we developed an approximate Schrodinger like equation to describe the finite M situation, but where there is no breaking of the heavy spin multiplets. The significant difference between the potential function of this equation obtained from the Skyrme profiles of the light pseudoscalars and the spherical

harmonic oscillator approximation to it was pointed out. This was more important for the $c-$baryons and for the excited states. The SHO approximation may be considered reasonable only in a rough qualitative sense. The general effect of going to finite M, but still infinite $m(N)$, was to reduce the strength of the binding. It was seen that the model including only light pseudoscalars was unable to give sufficient binding. With light vector mesons, however, the binding was substantially increased. At this level the excitation energy was predicted to be somewhat too low. Next, the effect of finite $m(N)$ was taken into account in a non-relativistic approximation. This further decreased the binding strength and favored one variant of the model with light vectors.

In subsequent work we plan to study the effects of using the Skyrme rather than the SHO potential (with light vectors) on the spectrum as well as on the wavefunctions (and Isgur-Wise functions). We would also like to further study heavy spin and $SU(3)$ breaking in the present framework.

Appendix A

Here we display the effect of multiplying the wavefunctions $\bar{\psi}_{dl}(g, g_3; r, k)$ (defined in (3.11)) by $(\hat{\mathbf{x}} \cdot \boldsymbol{\tau})_{ad}$, as required for the overall wavefunction given in (3.9). For simplicity we shall restrict ourselves to the case $g_3 = g$ and use the shorthand notation $\bar{\psi}_{dl}(g, g_3; r, k) = \Phi(r, k)$. Then

$$
\begin{aligned}
(\hat{\mathbf{x}} \cdot \boldsymbol{\tau}) \Phi(g, 0) &= \frac{-1}{\sqrt{2g+1}} \left[\sqrt{g}\, \Phi(g-1, 1) - \sqrt{g+1}\, \Phi(g+1, 1) \right], \\
(\hat{\mathbf{x}} \cdot \boldsymbol{\tau}) \Phi(g, 1) &= \frac{-1}{\sqrt{2g+1}} \left[\sqrt{g+1}\, \Phi(g-1, 1) + \sqrt{g}\, \Phi(g+1, 1) \right], \\
(\hat{\mathbf{x}} \cdot \boldsymbol{\tau}) \Phi(g-1, 1) &= \frac{1}{\sqrt{2g+1}} \left[-\sqrt{g+1}\, \Phi(g, 1) - \sqrt{g}\, \Phi(g, 0) \right], \\
(\hat{\mathbf{x}} \cdot \boldsymbol{\tau}) \Phi(g+1, 1) &= \frac{1}{\sqrt{2g+1}} \left[-\sqrt{g}\, \Phi(g, 1) + \sqrt{g+1}\, \Phi(g, 0) \right].
\end{aligned}
\tag{A.1}
$$

The wavefunctions in the first line, with $k = 0$, are the bound ones. The others, with $k = 1$, are unbound. Note that the wavefunctions on the first two lines both have parity $= (-1)^g$ and are manifestly orthogonal to each other. Those on third and fourth lines have parity $= -(-1)^g$ and are also manifestly orthogonal. In the special case when $g = 0$ there are just two, rather than four, independent states - $\Phi(1, 1)$ with positive parity which is bound, and $\Phi(0, 0)$ with negative parity which is unbound.

Appendix B

The equations of motion which follow cannonically from (5.1) are:

$$
-\mathcal{D}_\mu \mathcal{D}_\mu \bar{P} + M^2 \bar{P} - 2iMd\, p_\mu \bar{Q}_\mu = 0,
\tag{B.1}
$$

$$-\mathcal{D}_\mu \left(\mathcal{D}_\mu \bar{Q}_\nu - \mathcal{D}_\nu \bar{Q}_\mu\right) + M^{*2}\bar{Q}_\nu + 2iMd\,p_\nu\bar{P}$$
$$+2id'\,\epsilon_{\beta\nu\mu\sigma}\,p_\mu\,\mathcal{D}_\sigma\,\bar{Q}_\beta = 0 \ . \tag{B.2}$$

We choose to deal with $\bar{P}$ rather than P since $\bar{P}$ transforms like a light quark (rather than antiquark) and the interesting heavy meson dynamics is associated with its light constituents. (B.2) does not involve time derivatives of the component $\bar{Q}_4$. Hence $\bar{Q}_4$ is non-dynamical and may be eliminated in terms of the other fields. Correct to subleading $1/M$ order,

$$\bar{Q}_4 \approx -\frac{1}{M^{*2}}\mathcal{D}_i\,\partial_4\bar{Q}_i\,. \tag{B.3}$$

Making the ansätzae (5.4) for the ground state wavefunction and substituting into (B.1)-(B.3) yields, in addition to (5.6) the two additional equations:

$$-\psi_1'' - \frac{2}{r}\psi_1' + \left[M^{*2} - \omega^2 + \frac{2}{r^2}(1 + \sin^4\frac{F}{2})\right]\psi_1$$
$$+\sqrt{2}\left[-\frac{2\sin^2\frac{F}{2}}{r^2} + \frac{F'\sin F}{2r} + \frac{\omega d'\,\sin F}{r}\right]\psi_2 + MdF'\,\phi = 0 \ , \tag{B.4}$$

$$-\psi_2'' - \frac{2}{r}\psi_2' + \left[M^{*2} - \omega^2 + \frac{2}{r^2}(1 + 2\sin^4\frac{F}{2} - 2\sin^2\frac{F}{2}) - \omega d'F'\right]\psi_2$$
$$+\sqrt{2}\left[-\frac{2\sin^2\frac{F}{2}}{r^2} + \frac{F'\sin F}{2r} + \frac{\omega d'\,\sin F}{r}\right]\psi_1 - \frac{\sqrt{2}Md\,\sin F}{r}\,\phi = 0 \ . \tag{B.5}$$

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Figure Caption:

Figure 1: Graph of the effective potential (5.11) with (5.12) due to light pseudoscalar mesons. For comparison the graph of the quadratic approximation (5.13) is also shown.